Hydrostatic and Hydrodynamic forces

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Introduction.

In this lecture I restrict myself to the subjects mentioned in the title. In other lectures of this session of the congress we will learn much more about the practical applications of hydrodynamics in nature.
It is useful to subdivide the topic in the two parts:
- hydrostatic forces which are the forces in a fluid at rest and
- hydrodynamic forces which are the forces in a fluid in motion.

The contents of this lecture is based on material from various books. They are mentioned in the list of references in the next section. The first one (1) is advised for those interested in an important application related to this congress. Reference to the other books is made by means of the number in ().

General Liquid Properties.

As water is a liquid we have to consider the basic properties of a liquid first. Some of the properties are general, while others may be specific and related to the physical properties of the liquid, which itself is determined by its chemical composition.
There are two general properties, viz.
- a liquid has a fixed volume and differs from a gas and
- a liquid has fluidity and differs from a solid.

By the first property a liquid is different from a gas, but in both substances the particles can move relatively to each other. In a liquid the particles remain together because of the mutual attraction forces. The actual shape at rest is prescribed by internal or external conditions. An external condition is the shape of the containing vessel with the same volume. In a vessel which has a larger volume than the liquid or when the vessel is open on the upper side, the external influence of gravity will cause that the liquid seeks the lowest possible level. The most upper free surface of the liquid is then flat at rest and horizontal. You are familiar with this fact in daily life when using cups and communicating vessels.
In the absence of gravity an internal condition controls the shape of the liquid volume. It is the capillary attraction, which creates a surface tension. Through this effect the outer surface of the liquid tends to assume the minimum possible area, like the effect of a membrane. When there are no other external conditions the surface is spherical, which is true for free falling rain drops.
In het present lecture we have to deal with the effect of gravity, not with the effect of capillary attraction.

The fluidity property of liquids is also the reason for the alternative name; liquids are also named fluids. Fluids differ from solids; solids have rigidity or a permanent reaction against a change in relative positions of particles. Fluids do not have permanent reactions; a fluid in flowing motion has resistance against changes but this resistance disappears when the fluid comes to rest. So in a state of rest the fluidity property is perfect. In a flowing fluid viscosity gives rise to internal friction. This effect depends on the flow rate.
An alternative way to indicate the difference between fluids and solids is the difference in elasticity properties. This is mentioned here for completeness sake. Solids have form elasticity and bulk elasticity. Fluids have no form elasticity, which is the effect described above. Fluids only have bulk elasticity or compressibility. This means that fluids have even at rest a reaction against change in volume. Practically, however, fluids are said to be incompressible, which is certainly true for water in the context of this congress. It is worth noting that as a consequence of the small compressibility of water, sound travels relatively fast in water, i.e. more than four times faster than sound travels in air.
In general the volume of an amount of fluid depends on the temperature and pressure. The latter quantity has to be given much more attention in the context of hydrostatic and hydrodynamic forces.

Hydrostatic Forces in a Fluid at Rest.

Topics:

Pressure:

1. Hydrostatic Pressure
2. Measures of Pressure
3. Hydrostatic Paradox and Pascal's law

Force:

4. Buoyancy and Archimedes' law
5. Equilibrium and Stability
6. Floating Bodies

Hydrodynamic Forces in a Fluid in Motion.

Topics:

Flow:

7. Velocity and stream lines
8. Streamline patterns
9. Curved streamlines and deformation
10. Relation between velocity and pressure
11. Shear flow and flow stability
12. Laminar and turbulent flow

Bodies in flow:

13. Boundary layers and wall friction
14. Flow separation and effect on drag
15. Lift forces and measures to create lift
16. Drag forces on the human body

1. Hydrostatic pressure.

In a fluid at rest on earth the gravitational force exerted by this globe plays a dominant role. The static pressure in a fluid is built up by gravity together with the fluid's own ponderosity.

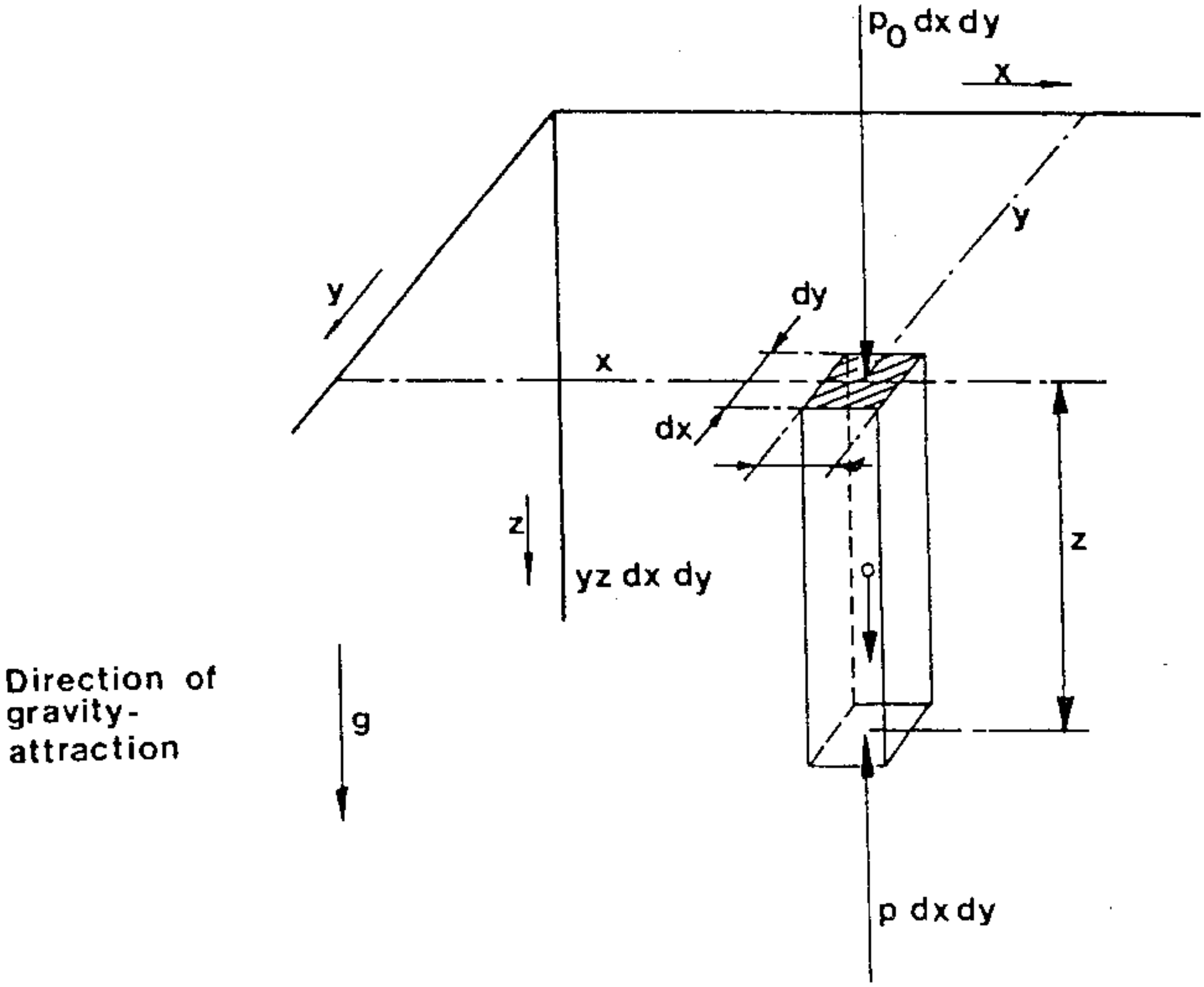


Fig. 1 : A column of fluid gives pressure, (6)

At a point in a fluid we can think of a small horizontal surface on which a force is exerted. Vertically above that surface there is a column of water and air. The total weight of that amount of water and air is the force which defines the static pressure. The total weight is proportional to the surface area, so we can speak of a force per unit of surface area and that is called pressure.
The fluid weight is proportional to its mass density, i.e. its mass per unit volume. Therefore also the hydrostatic pressure in a fluid depends on its mass density. Small differences in mass density can have consequences through its effect on the hydrostatic pressure. Familiar examples are buoyant flows in a water kettle being heated and the difference in floating on sea water and fresh water. The subjects of buoyancy and floating will be dealt with more extensively.

2. Measures of Pressure.

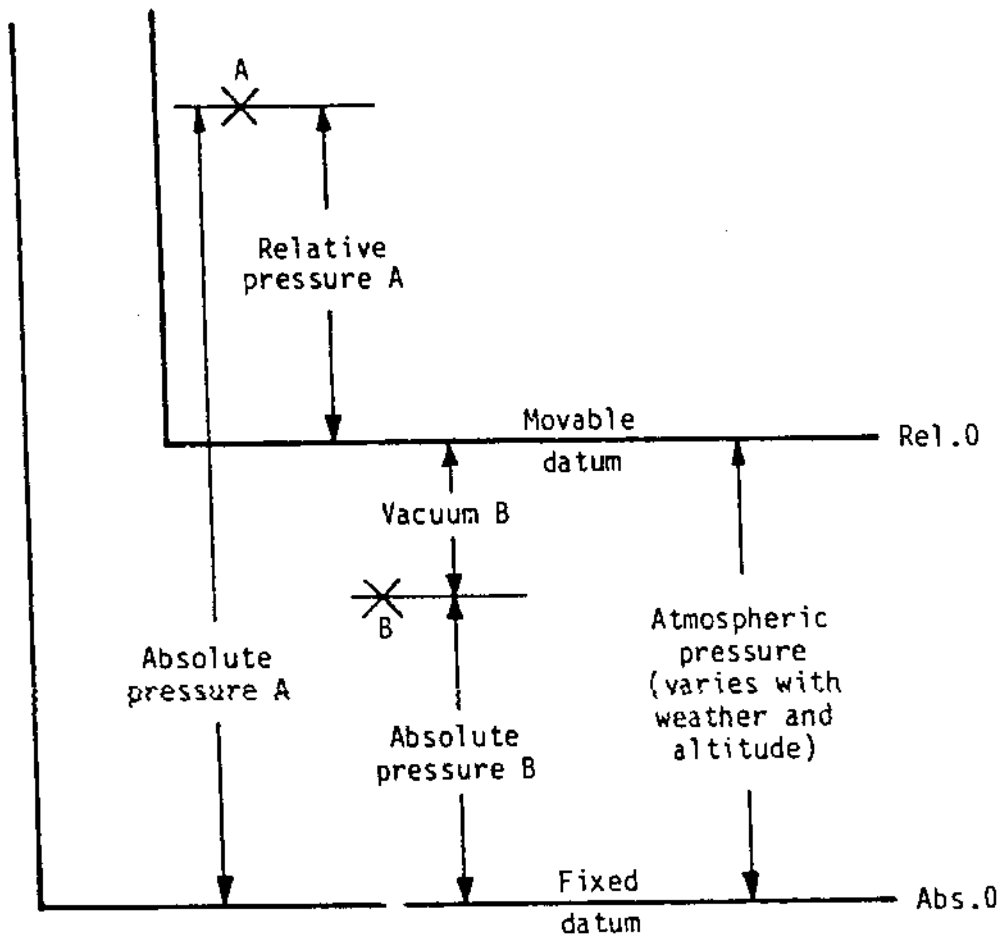


Figure 2 : Various pressure levels, (7)

The official measure for pressure is one Pascal (Pa), which means a force of one Newton on a surface of one square meter. 1 Pa is a small quantity if compared with the atmospheric

pressure in which we live. In the atmosphere at the free surface level of the sea the pressure is about 100,000 (one hundred thousand) Pa. This pressure level is called one bar. The pressures which play a role in swimming are much smaller.
Instruments by which the pressure is measured are usually calibrated from the atmospheric pressure level. They indicate the relative pressure and are set zero in the atmosphere. So under water they measure the hydrostatic pressure only. From the definition it follows that the hydrostatic pressure is proportional to the depth of submergence. For heavier fluids (more weight) the increase of pressure with depth is stronger than for lighter fluids.

3. Hydrostatic Paradox and Pascal's Law.

The understanding of the hydrostatic pressure seems easy by thinking of the column of water above the point of observation. However, there are some special effects which do not follow directly from this concept but have to do essentially with the property of fluidity.

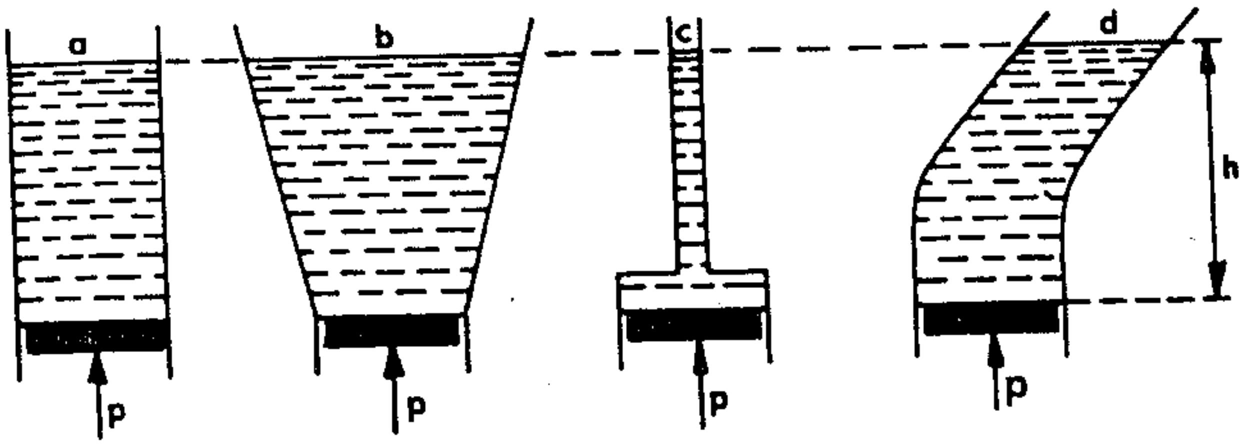


Figure 3a : Pressure on the bottom of a vessel, (6)

First there is the socalled hydrostatic paradox: If we have vessels with different shapes but the same bottom area, the pressure on these bottoms is the same if the vertical distance to the free water surface is the same. The amount of liquid in the vessel does not matter. The force on the bottom can be much higher than the weight of the liquid in the vessel. We can conclude that the idea of pressure is much wider than just the weight of a column of fluid, which I used as a means to introduce the subject.

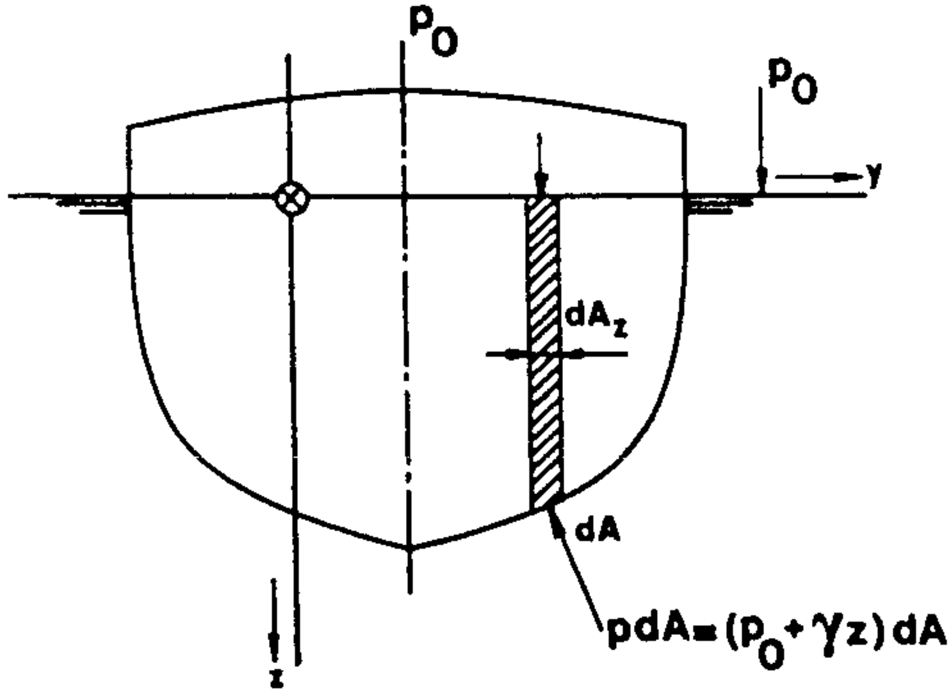


Figure 3b: Pressure on the hull of a body (6).

Second there is the law of Pascal, which states that the pressure at a certain submergence level is the same in all directions. The force on a surface is the same for all orientations. This law follows from the requirement that in a fluid in a state of rest all fluid elements must be in equilibrium. Then the forces on all sides of a small cube must be the same.
The picture is that of a floating ship, which is a body not too different from the subject on this congress. It has to be noted that the pressure on the hull is equal to the indicated column of water irrespective whether the column of water is there in reality or only in our imagination.

4. Buoyancy and Archimedes' Law.

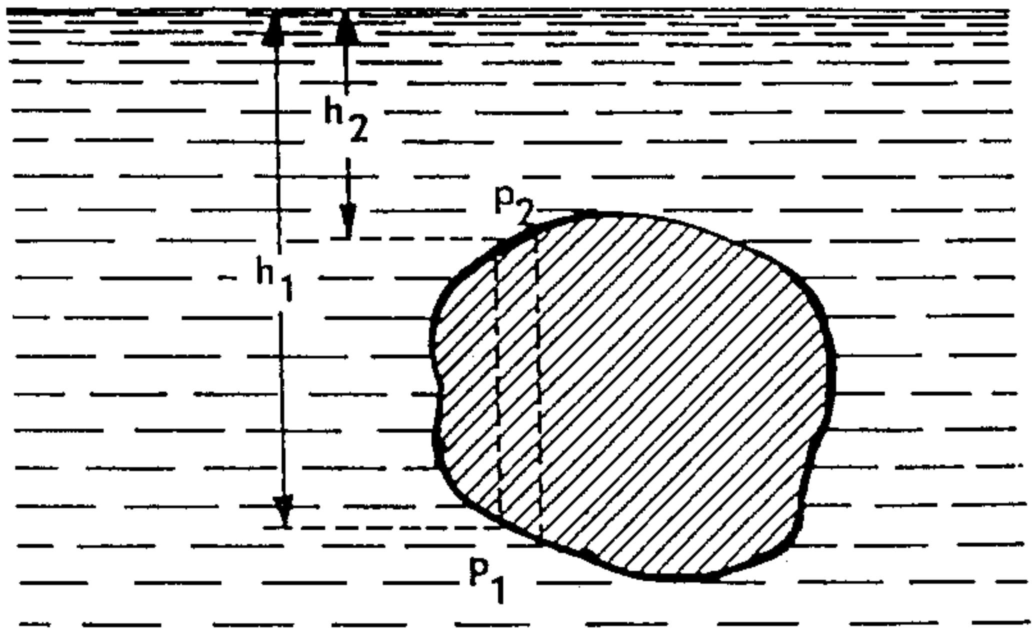


Figure 4: A fully submerged body (6).

The forementioned properties of hydrostatic pressure are the base for understanding buoyance forces. Most familiar is the force on a fully submerged body which is an upthrust of the water on the body equal to the weight of the displaced volume of water. This is known as Archimedes' Law. More generally, pressures on the hull of a ship or on the skin of a body, when they are put together become forces on the ship or on the body. The force may have any direction depending on the shape of the body and on the pressure distribution. This will become clear in the sections on hydrodynamic forces when there is also flow around the body.

5. Equilibrium and Stability.

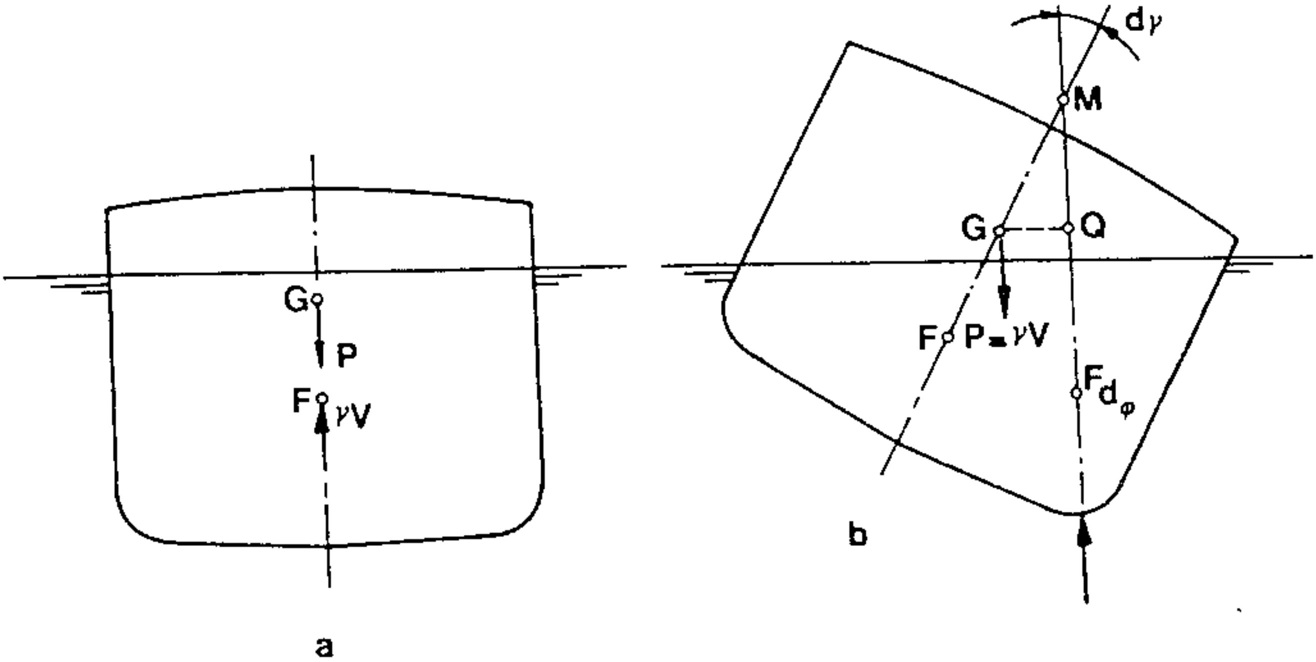


Figure 5: Partly submerged body in two positions (6).

When a body is not fully submerged the situation of hydrostatic force on the body is more complicated. But the introduction of forces instead of pressures is helpful. The body which is floating on water has its own weight, a force P acting in the centre of gravity G. For equilibrium it is necessary that the upthrust or buoyance force is equal to the weight, but the centre of this force is at a different location F. For a symmetric body like the ship in the picture on the left F and G are on the same vertical line.
The situation becomes more complicated when the body is not symmetric or not in a symmetric position like the ship in the picture on the right.
When the ship has rotated from its original upright position, the centre of gravity G remains the same. (Note that it is possible that the load inside the ship moves, e.g. oil in a

tanker, then also G may change its position relative to the ship.) The centre of buoyancy will change in general, say to Fd. The original line through F and G and the vertical line through Fd intersect at M. This point is called the metacenter. Now we can see that the gravity force pulls down the ship and the buoyance pushes up the ship in such a way that the original position of the ship is obtained again. This stabilizing effect is related to the relative position of M and G. When M is below G the ship is not stable.

6. Floating Bodies.

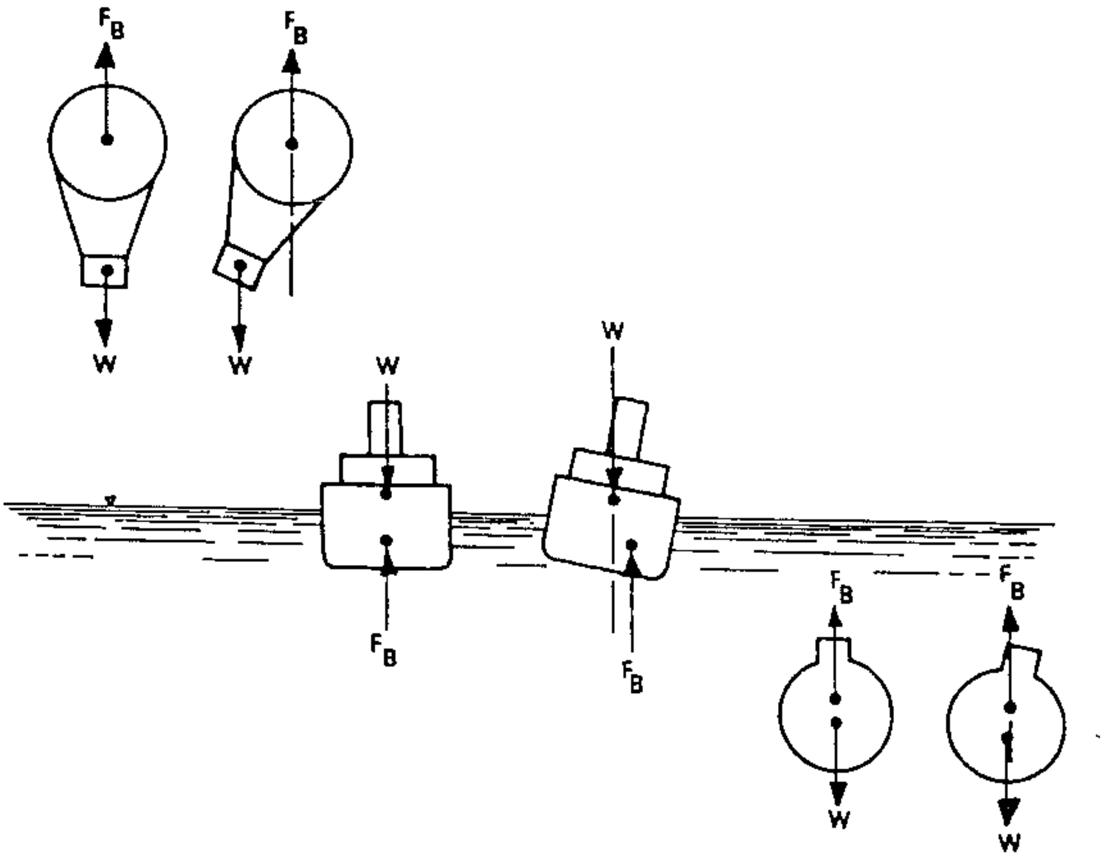


Figure 6 : Floating bodies, (7).

In this Figure you see three types of floating bodies, a balloon in the air, a submarine in the water and a surface ship at sealevel. All pictures show stable conditions, but it is important to note the different distances between W (weight) and Fb (buoyance). Also note that the balloon and the submarine both have fixed positions of the buoyance force.
The balloon is most stable through the lower position of basket with weight. The surface ship has not such a low position of the weight and is only stable thanks to the flexible position of the buoyance force. The submarine has not the stabilizing features of the balloon or the surface ship; now stability strongly depends on the position of Fb being higher than the position of W. The small distance between both

indicates that stability is weak in such a case of a fully submerged body.

7. Velocity and Streamlines.

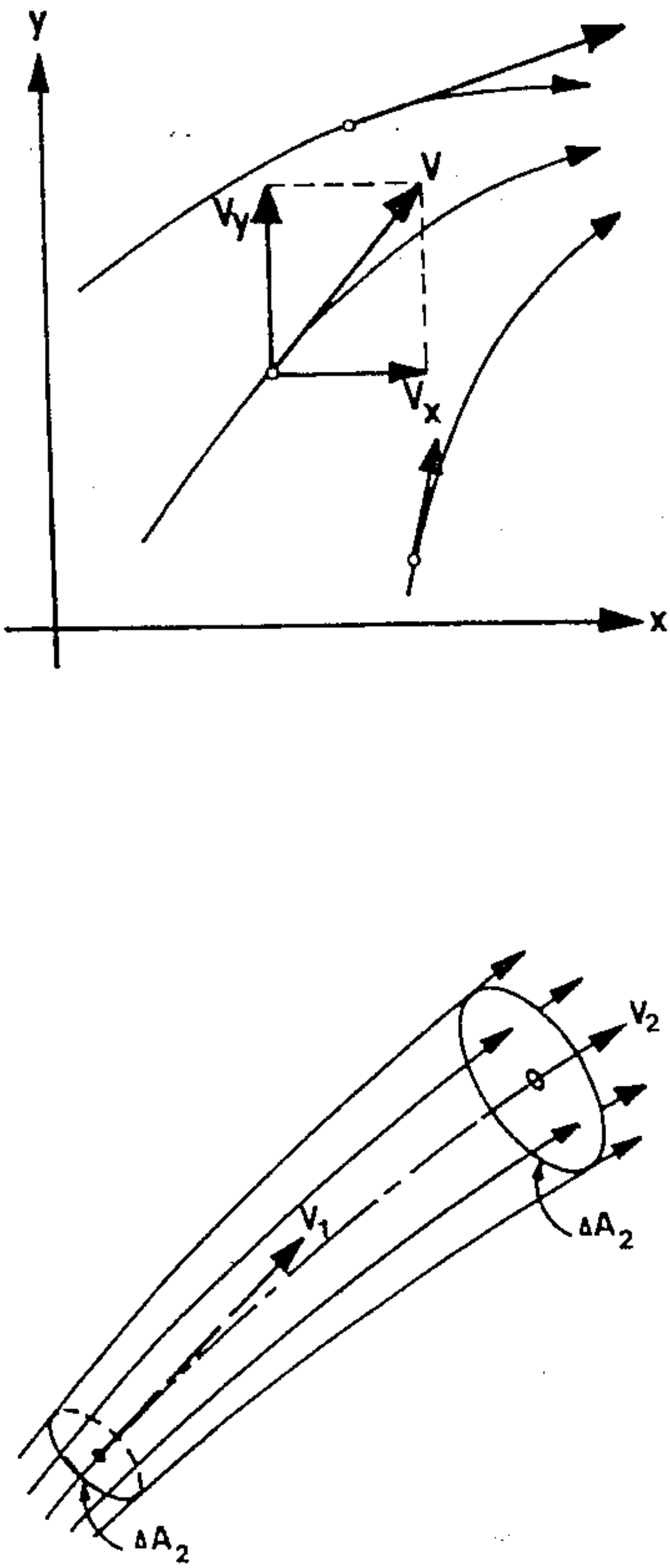


Figure 7 : Streamlines and velocity vectors (top); elementary streamtube (bottom), (5).

To understand different types of fluid motion some basic features have to be introduced. Each of these is directly related to the variation of the velocity of flow either with time or with location in the field of motion. Velocity is a vector quantity, since it possesses both magnitude and direction. The velocity vector at any point in a moving fluid therefore generally has components in each of the three directions of our space and, again as a general rule, each of these components may be expected to vary both from instant to instant and from point to point. A streamline is defined as a line which lies in the direction of flow at every point at a given instant; hence (see bottom picture) at any point the velocity vector and the stream line passing through that point must be tangential to one another, while the velocity component normal to the streamline at the point of tangency must necessarily have a zero magnitude.
If the velocity vector does not change in either magnitude or direction with passage of time at any fixed point in a moving fluid, such a flow is called steady. In unsteady flow either the magnitude or the direction of the velocity, or both together, will vary with time.
An elementary flow passage bounded by streamlines (see bottom picture) is known as a streamtube. It is evident that no flow passes through the walls of a streamtube. If the fluid is incompressible, it follows that at any instant the rate of flow past all successive cross sections of the stream tube must be the same. Therefore a convergence of stream lines will correspond to an increase of the velocity magnitude with distance and vice versa. This is called the law of continuity.

8. Streamline Pattern.

A pattern of stream lines reveals at a glance the direction and relative magnitude of the velocity throughout the field of flow.

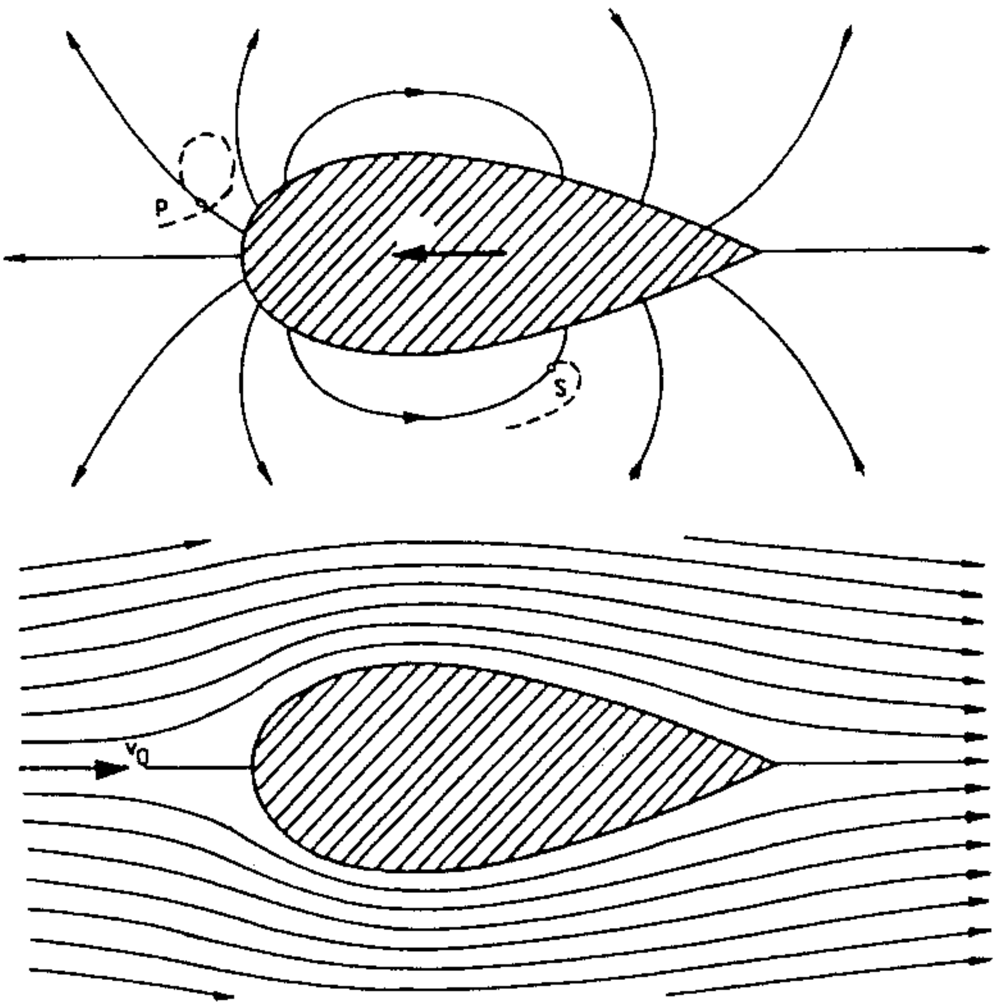


Figure 8 : Streamline patterns of flow around body, (5).

The flow around a body which moves in still water is an unsteady flow. If we observe such a flow from a fixed position, we see particles moving away from the body in the front region and particles moving towards the body in the aft region as illustrated in the top picture. If the body has a constant speed it is possible to transform the streamlines into that of a steady flow by use of the principle of relative motion. That occurs if we observe the flow from a position which moves with the body. What we see then is shown in the bottom picture. This procedure is the basic principle of watertunnel and windtunnel experiments. Herein bodies which normally move at constant speed through a fluid at rest, e.g. like ships, airplanes and swimmers are held stationary in a stream of fluid moving at the same relative speed. In this situations all kinds of measurements and visualization of flow become much easier.

9. Curved Streamlines and Deformation.

Now we want to have a closer look at what happens more precisely in the flowing fluid.

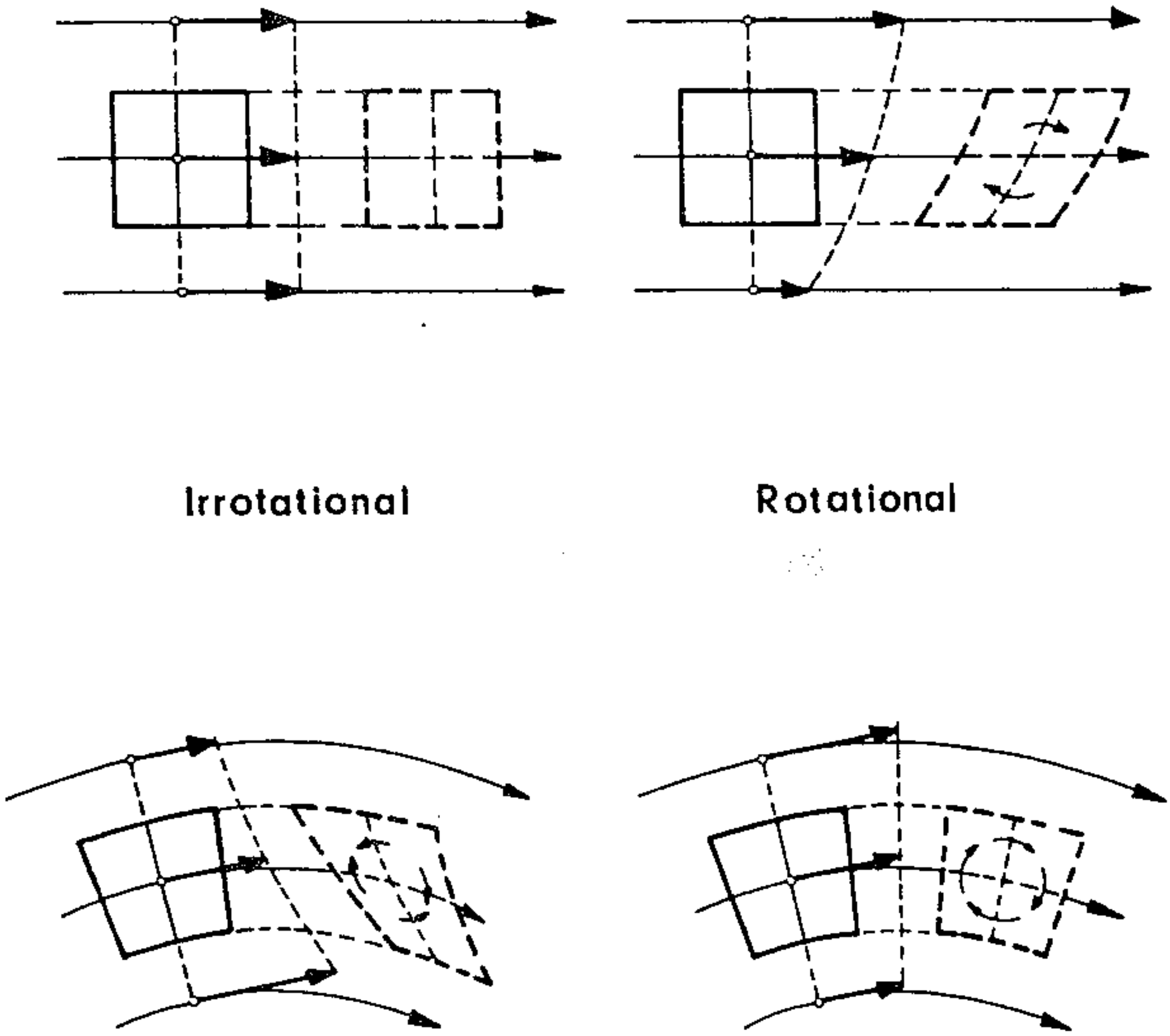


Figure 9 : Types of motion of fluid elements, (5).

Two basic motions of an elementary fluid element can be distinghuished, viz. motion along a curved streamline and deformation. In the upper pictures the effect of deformation in a flow with straight streamlines is shown. This effect depends on the difference in velocity along adjacent streamlines. In the top right picture we see that a fluid element is deformed and rotated. In the lower pictures the same effect is shown for a flow with curved streamlines. On the left the difference in velocity between streamlines is such that the fluid element is strongly deformed, but as a whole not rotated. Therefore this is still called irrotational flow. On the bottom right picture the difference in velocity between streamlines is such the fluid element is not deformed but rotated as a whole. From this illustrations we see that fluid elements can undergo different treatments. This can have important consequences for the description of the flow by using mathematics, but that will not be done here.

10. Relation between Velocity and Pressure.

Having discussed now the kinematical aspects of a fluid in motion, viz. its velocity distribution and its streamlines, we have to return to the pressure. The pressure distribution has to be understood in order to learn about hydrodynamic forces similarly as hydrostatic forces.

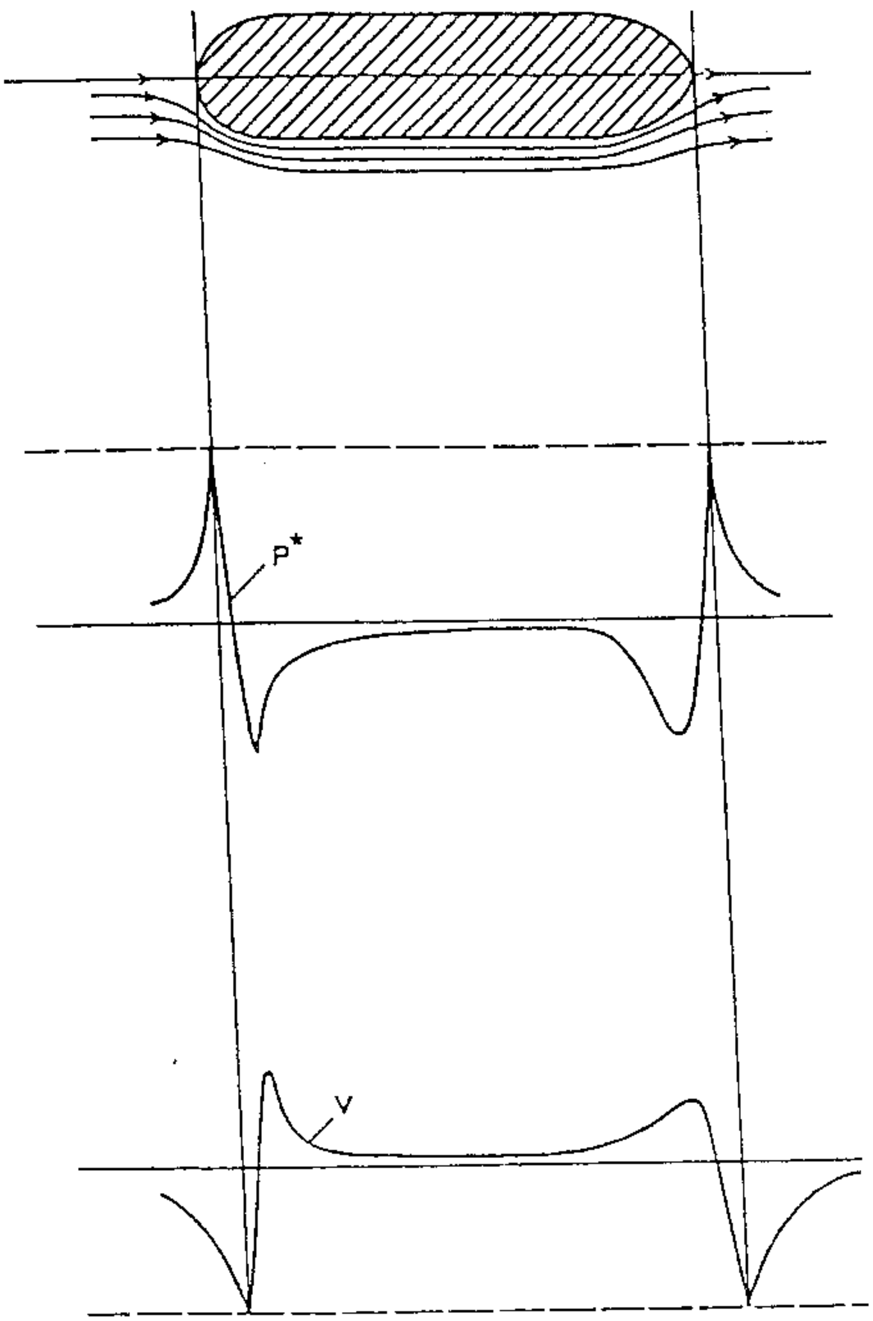


Figure 10 :

Velocity and Pressure distribution on a body.

If we could follow the flow along a streamline we would experience that at places of low velocity the pressure is high and vice versa. In the picture this is demonstrated with graphs for pressure (P) and for velocity (V). Remember that streamlines are wider away in a low velocity region and close together at higher velocity.
This velocity-pressure relationship, which is called Bernoulli's law, is very important and represents major effects in the flow. However, it is not accurate for prediction of a force on a body. For that purpose we also take into account the friction on the body surface. As long as we disregard friction we encounter the socalled Paradox of d'Alembert, which says the force on a body in a steady stream is zero. That is contrary to intuition.

11. Shear Flow and Flow Stability.

In a flow we have to deal also with instabilities. This means that above a certain speed the streamlines loose their regular pattern. They become very complicated and can no longer be visualized.

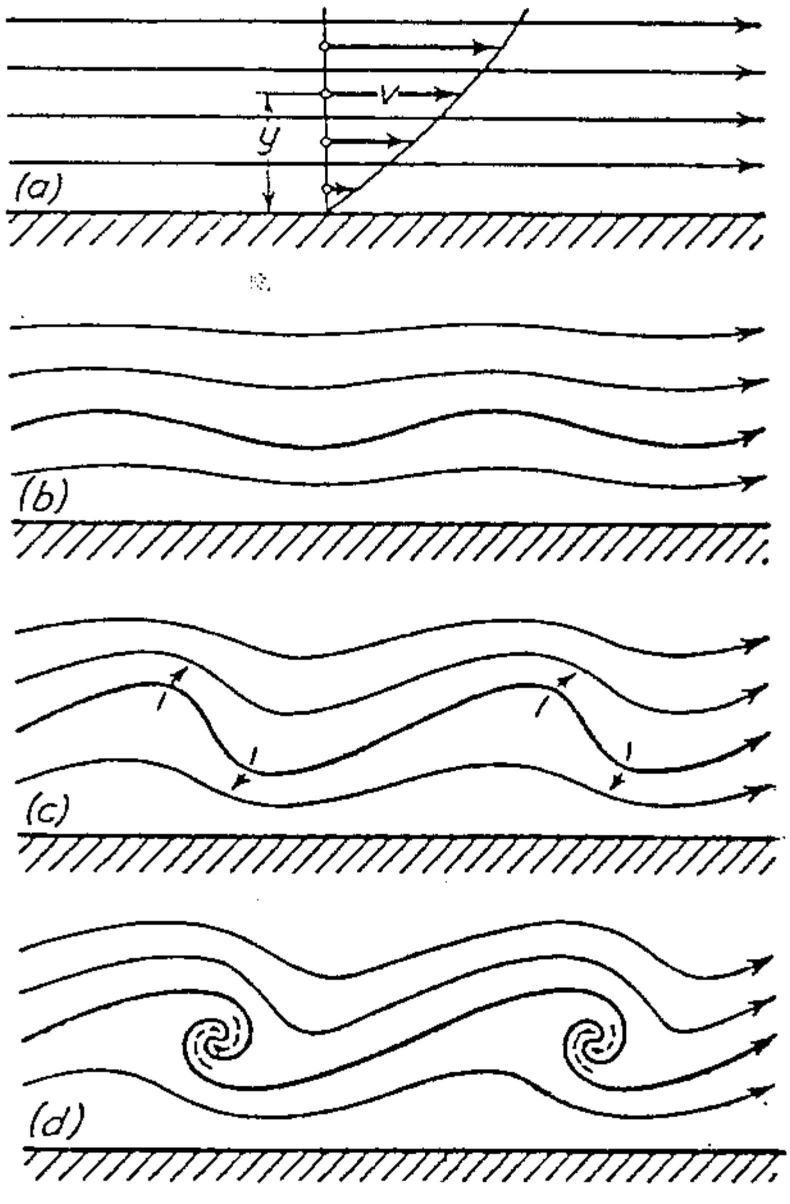


Figure 11 : Growth of a disturbance in a flow, (5).

The underlying mechanism is by no means simple, but a qualitative analysis may be helpful in understanding. Assume that a laminar flow takes place parallel to a plane boundary (picture a). Such a flow has zero speed at the wall due to the effect of viscosity which prevents slip along the wall. The flow appears as layered with increasing speed away from the wall. At some distance from the wall a disturbance is produced which yields at a given instant local undulations of the streamlines (pictures b). The accompanying variation in the velocity will give rise to a pressure pattern which tends to augment the undulations (pictures c and d). On the other side there are effects from viscosity and of the presence of the wall which may dampen the undulations. In practical situations it depends on the flow configuration whether the original flow is stable or not.

12. Laminar and Turbulent Flow.

Laminar flow is defined as that in which the streamlines remain distinct from each other over their entire length. Laminar flow may be either steady of unsteady and either uniform or nonuniform. Turbulent flow is the opposite of laminar flow; once the heterogeneous mixing process started by flow instabilities exists, even the instantaneous streamlines become thoroughly confused. Strictly speaking turbulent flow is inherently unsteady and nonuniform, but it is convenient to distinghuish between the secondary motion of the turbulence and the primary motion of the fluid. The latter can be classified as either steady or unsteady, uniform or nonuniform and rotational or irrotational without regard to the turbulence itself.

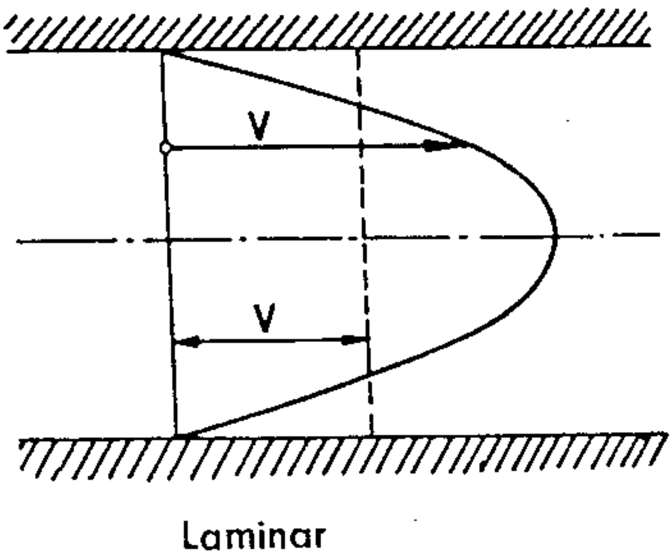


Figure 12a : Laminar flow in a duct, (5).

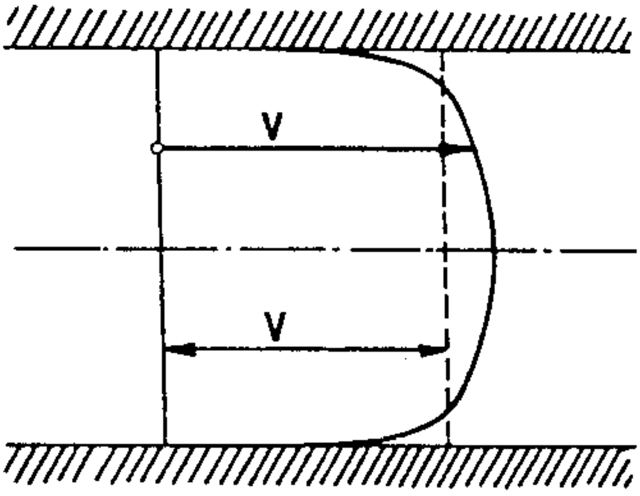


Figure 12b : Turbulent flow in a duct, (5).

The effect of turbulence is a lateral mixing process which results in an equalization of the mean velocity vector across the flow section.

13. Boundary layer and wall friction.

As real fluid flows over the surface of a body, the effect of viscosity is first that the fluid sticks to the surface by adhesive forces. But away from the surface the fluid tries to follow the main stream. The laminar flow is in a layer of varying thickness; it begins from no thickness at the front of the body where the fluid first contacts it and increases along the surface in the direction of the motion. Such a laminar boundary layer may change into a turbulent boundary layer.

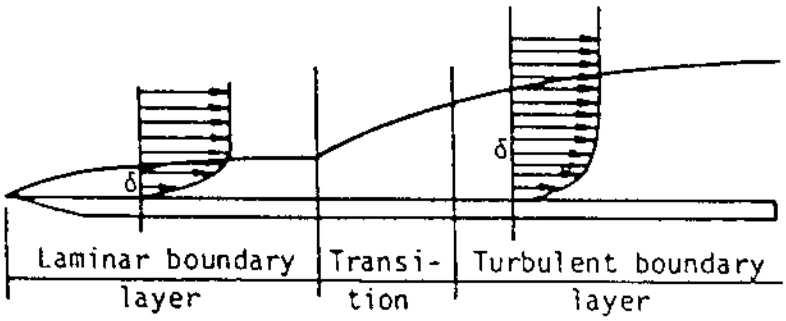


Figure 13 : Boundary layers along flat plate, (7).

In the picture these phenomena are illustrated for a simple body, a smooth flat plate. The boundary layers occupy an extremely thin and usually invisible part of the flow (the thickness is exaggerated in the picture). Nevertheless it is of great importance since it is the essential reason for the existence of a frictional drag force exerted by the fluid on the body. The drag force also depends on the quality of the surface, smooth or rough.

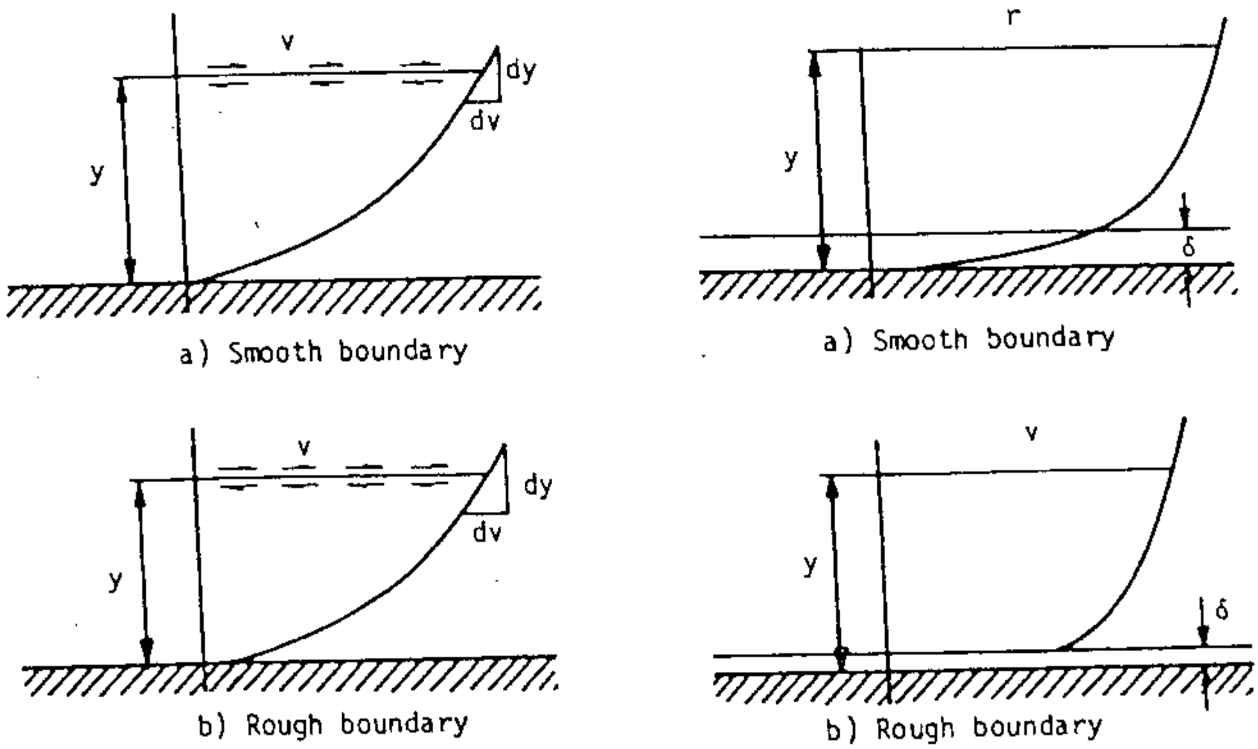
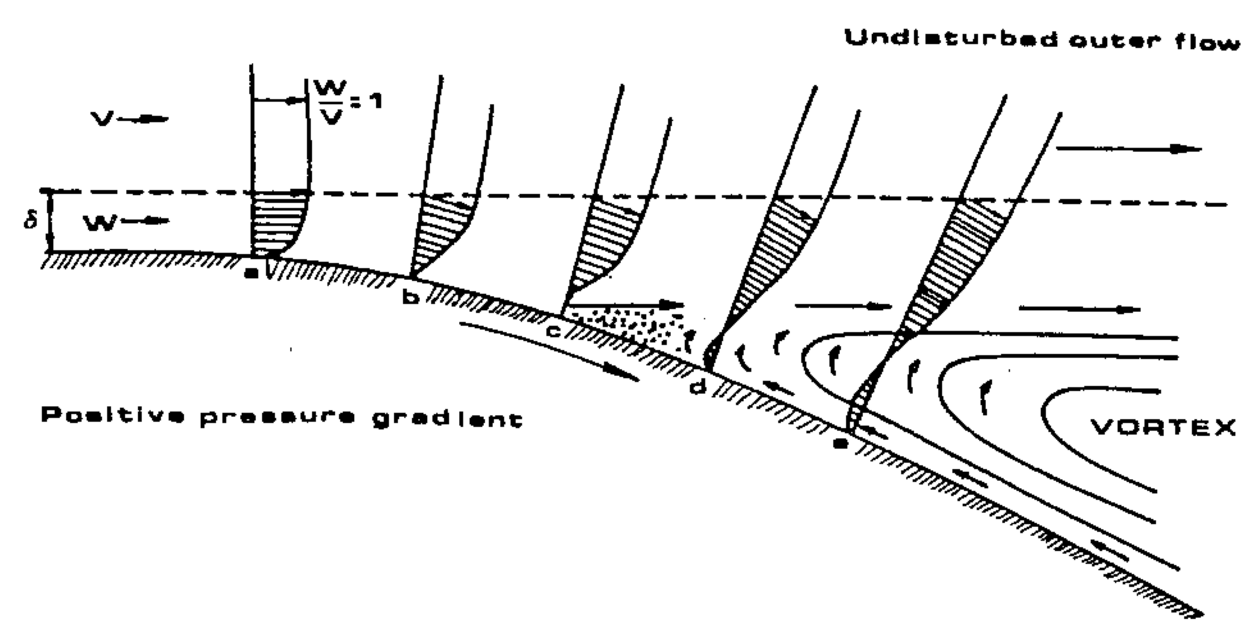


Figure 14 : Smooth and rough surfaces, (7).

Laminar flow occurring over smooth or rough surfaces possesses the same properties in both cases, as depicted in the pictures (a) and (b) on the left. Thus in laminar flow, surface roughness has no effect on the flow.
In turbulent flow, however, the fluid motion is influenced. When turbulent flow occurs over smooth solid boundaries it is always separated from the boundary by a film of laminar flow. This can be explained as follows. The presence of a boundary in a turbulent flow will reduce the freedom of the turbulent mixing process and in the region very close to the boundary the flow remains laminar. Now a rough boundary of a turbulent flow appears as smooth as long as the roughness particles or obstacles are completely submerged in the laminar film. However, when the height of the roughness elements exceeds the thickness of the laminar film the turbulence will be augmented and the laminar film is no longer effective. A different velocity profile of the flow results as depicted in the pictures (a) and (b) on the right.

14. Flow separation and effect on drag.

We have looked at a number of details of the flow, but now we will pay attention to a broader view of bodies in motion in a flow. Much of the hydrodynamic forces can be understood by the effect of separation.



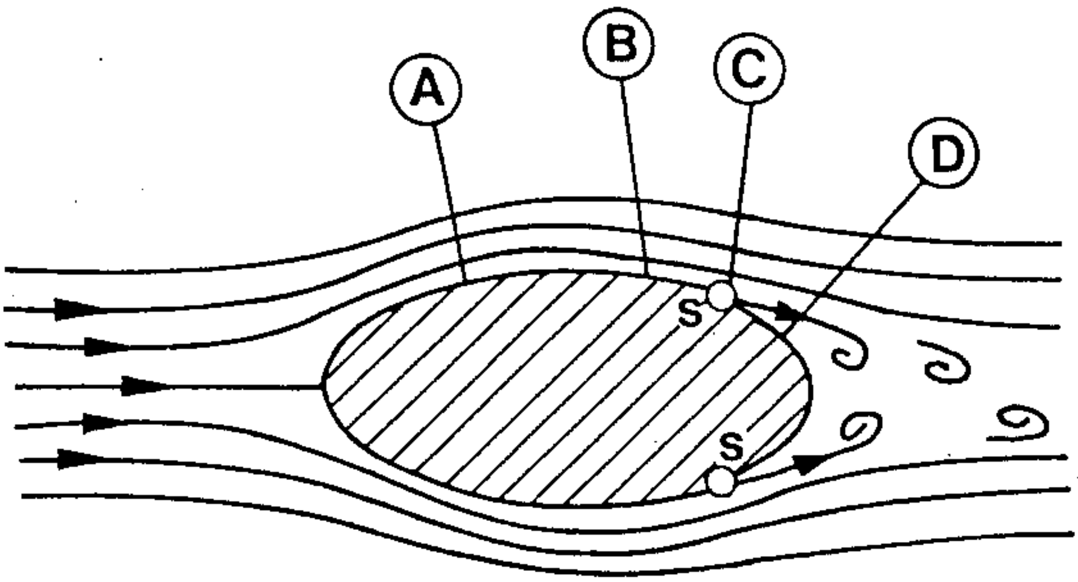
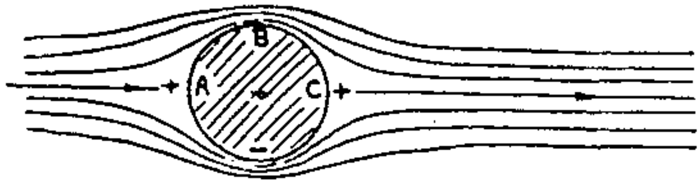
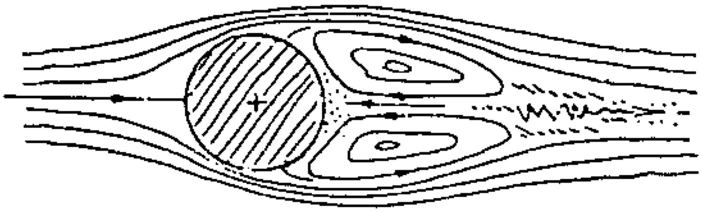


Figure 15 : Separation of flow at aft end of body, (3).

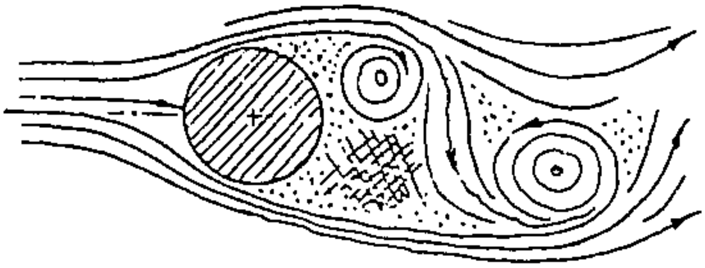
The fluid elements in the boundary layer loose energy and momentum in comparison with fluid elements in the outer flow. Near the aft end of a body the pressure tends to increase. As a result the fluid particles from the boundary layer can not move further; they come to rest, accumulate and are given a rotary motion by the main stream. This rotation can grow and an eddy of increasing size is developed. The eddy cannot be retained in the presence of the body but breaks away. Then another is allowed to form and the process repeats itself.



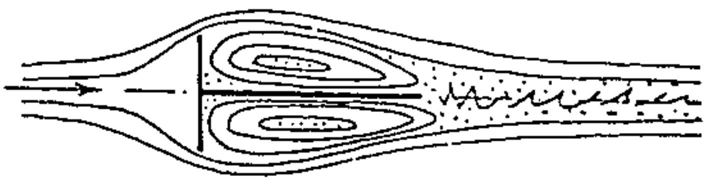
(A) FLOW PATTERN OF CIRCULAR CYLINDER IN NON'VISCOUS FLOW; NO DRAG.



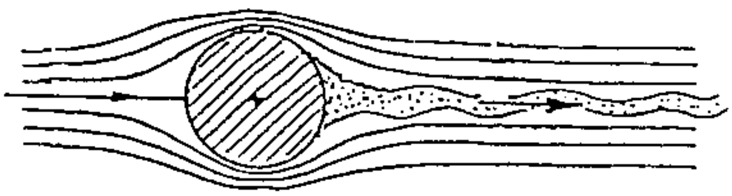
(B) CYLINDER AT REYNOLDS NUMBERS IN THE ORDER OF 40; $C_D \approx 1.2$.



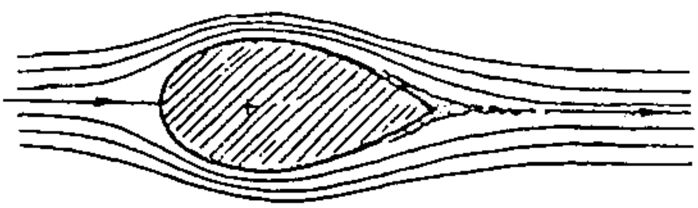
(C) CYLINDER BETWEEN $R_d = 10^4$ and 10^5; VORTEX STREET WITH $C_D = 1.2$.



(D) PLATE WITH "SPLITTER" DEVICE IN WAKE; $C_D = 1.5$.



(E) CYLINDER ABOVE CRITICAL REYNOLDS NUMBER WITH $C_D = 0.3$.



(F) STREAMLINE SECTION WITH C_D IN THE ORDER OF 0.06.

Figure 16 : Separation and turbulent wakes, (3).

The result of separation and eddy formation is the formation of a turbulent wake. The turbulence in the wake is of other nature than in a boundary layer. Many situations can be seen in the picture.
To return now to the problem of drag forces on objects in a fluid flow, the effects of boundary layers, separation and wakes may be observed. Fundamentally drag is caused by the components of the normal and tangential forces transmitted from the fluid to the surface elements of the body. The normal forces are those of pressure which in general may be calculated by applying Bernoulli's law to the streamtube adjacent to the body. The tangential forces are those of shear at the surface of the object arising from viscous effects in the boundary layer.

15. Lift Forces and Measures to Create Lift.

Based on considerations of the effect of viscisity, laminar flow and turbulent flow, we have discussed the resistance of bodies moving in water. This force acts in longitudinal direction i.e. in the direction of the flow. There is also a force perpendicular to this direction. It is called the lift force and is of great importance in the hydrodynamic propulsion of bodies.
The lift force also is a result of the pressures around a body. To obtain lift it is necessary that in the flow a circulation is created. To understand this effect it is useful to think of long bodies like airplane wings. The longest dimension is not in the direction of the flow but perpendicular to that. The other dimensions are relatively small. This facilitates that we can discuss the properties of the flow in a plane parallel to the flow. The flow in adjacent planes does not differ from each other.

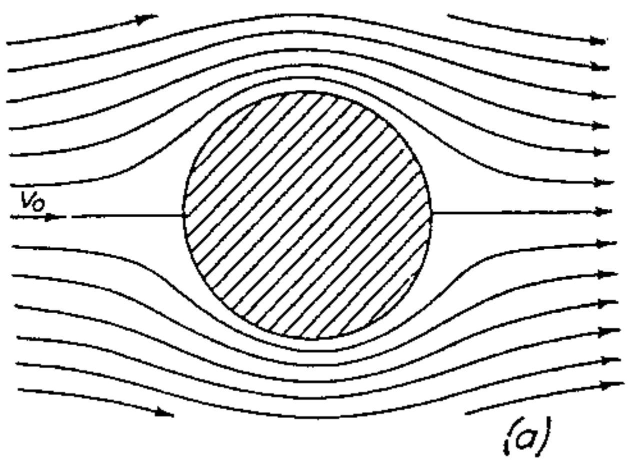


Figure 17a : Circulatory flow creating lift, (5).

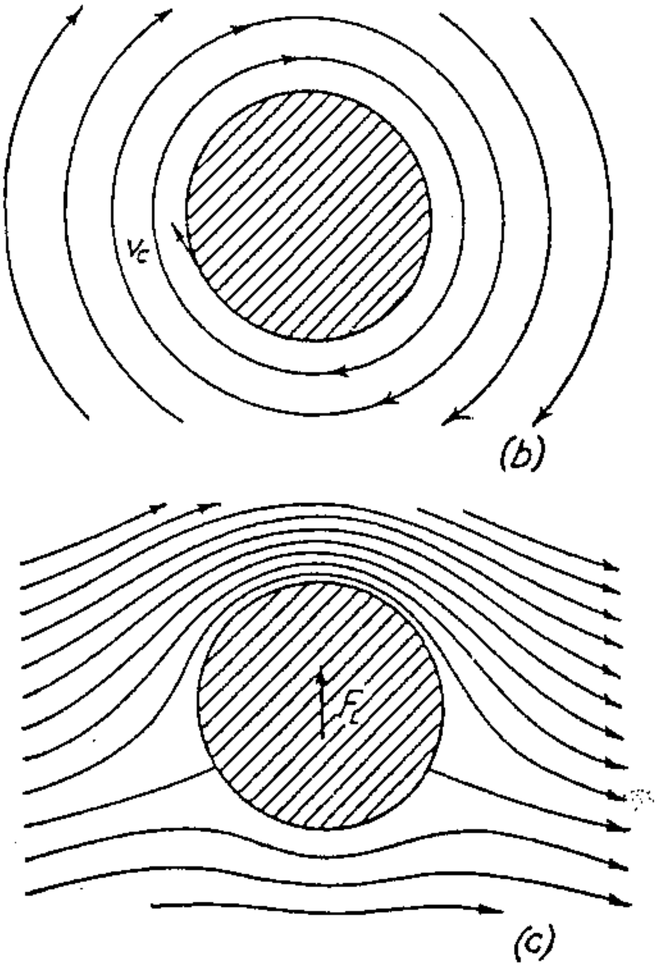


Figure 17b, 17c : Circulatory flow creating lift, (5).

In the picture streamlines around a cylinder are considered. The effect of boundary layers is disregarded. Although boundary layers play a specific role in the creation of circulatory flow its effect on the magnitude of the lift in relation to circulation is small. (Hence we can use Bernoulli's law). In picture (a) there is no circulation; only an approaching flow from the left. In this symmetric flow pattern there is no reason to expect a vertical force on the cylinder as pressures are equal on both sides.
The situation becomes different when the flow is combined with a pure circulation (picture (b)), leading to the flow pattern in picture (c). Far away from the cylinder the approaching flow is still uniform as the velocities of the circulatory flow are small there. Close to the cylinder, however, the flow pattern is strongly distorted in comparison with picture (a). The flow over the top has higher velocity than the flow along the bottom. As a consequence of the law of Bernoulli the pressure at the top is lower than at the bottom and there is an upward force called lift.

The lift force can be very strong in comparison with the drag force and it is very useful. In nature flying and swimming depend on this phenomenon, while it is utilized by men in sailing, ship propulsion, pumping, windmills and again flying. It will be clear that by some means we have to control the circulation around the body.

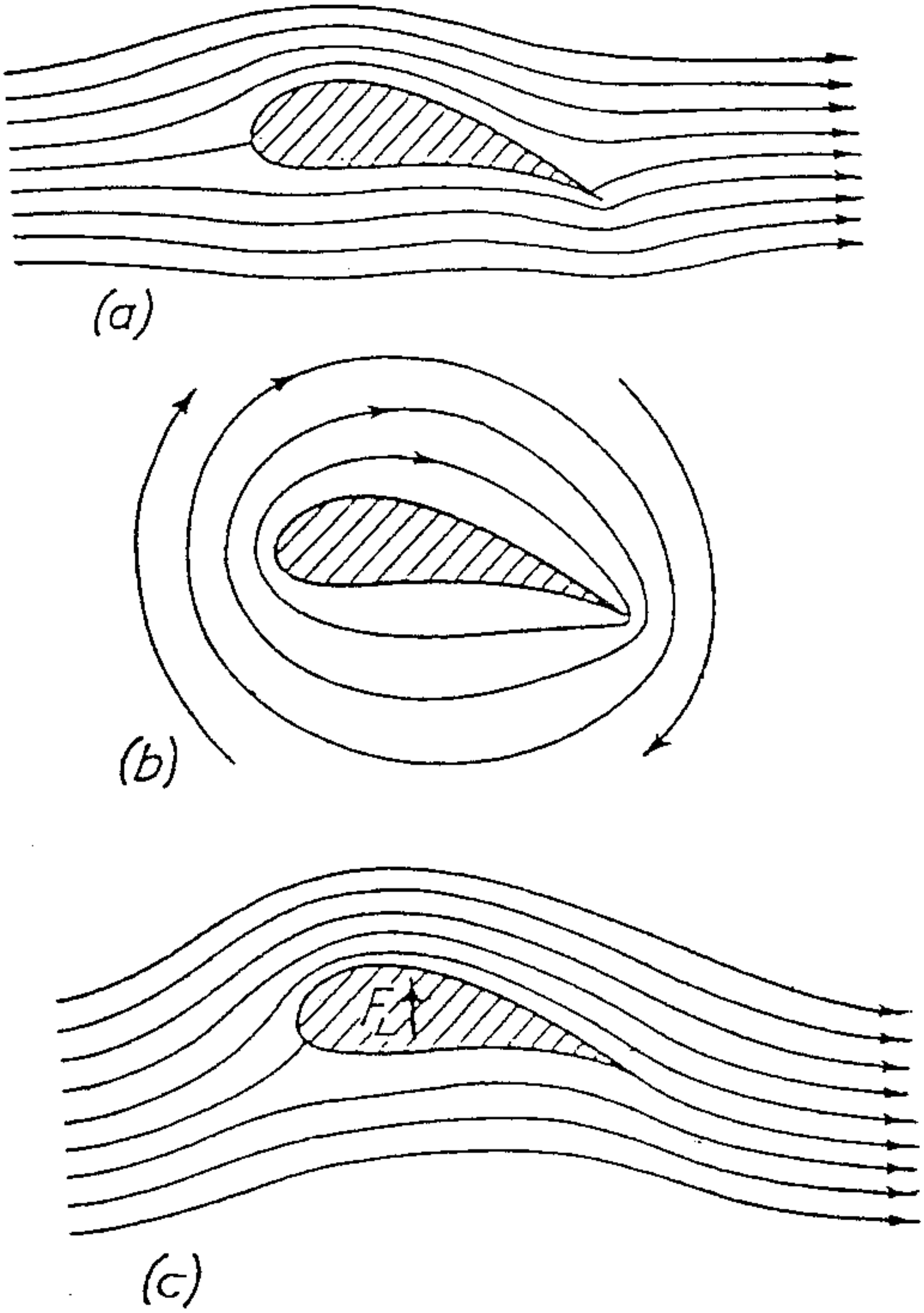


Figure 18 : Profile which creates circulation, (5).

The difference between the body in this picture and the previous one is clear. It is not a cylinder but a profile. The most important feature of a profile is that the streamlines at the trailing edge follow the body contour as smooth as possible. This is seen in picture (c) which is different from (a). The difference is due to boundary layer effects, which create the required circulation, picture (b), to make the flow smooth. As a consequence there is lift.

16. Drag Forces on the Human Body.

To conclude this lecture we show some practical information about hydrodynamic forces on a human body when it is placed in a stream in various positions. The results are obtained from measurements in a windtunnel, so strictly we have to speak of aerodynamic forces. But the results are presented in such a way that they also apply to circumstances in water.

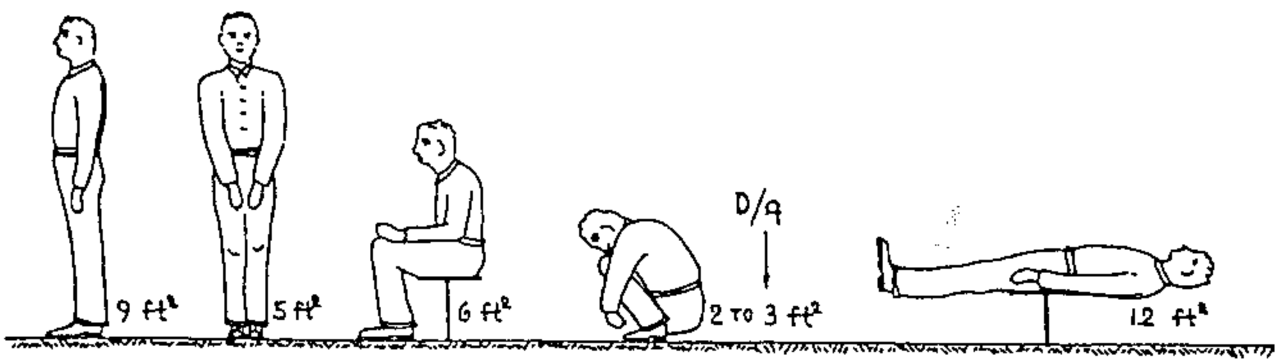


Figure 19 : Human body in a constant stream, (3).

The human body is of the shape similar to a cylinder of a lenght to diameter ratio between 4 and 7. Cylinders are of great technical interest and therefore a useful reference. The drag of the human body is expressed as an effective cross section which is confronted with the flow. This cross section is calculated from the measured drag D and from the reference pressure q of the flow. q is proportional to the mass density of the fluid multiplied by the velocity squared.
The cross sectional areas in the picture are given in square feet. The given numerical values are effective in regard of drag; they may differ from the real cross sectional areas in regard of the geometry. As a reference it can be mentioned that the total surface area of this human body is 20 square feet.

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